Worked Solutions

Edexcel C4 Paper B

1. (a) $xy - 2y + 5x - 10 = 12$

$$x\frac{dy}{dx} + y.1 - 2\frac{dy}{dx} + 5 = 0$$

$$\frac{dy}{dx}(x-2) = -(y+5)$$

$$\frac{dy}{dx} = \frac{y+5}{2-x}$$ **(2)**

(b) at $(4, 1)$, $\frac{dy}{dx} = \frac{1+5}{2-4} = -3$

$\therefore$ gradient of normal $= \frac{1}{3}$

equation of normal is $\quad y - 1 = \frac{1}{3}(x-4)$

$$3y = x - 1$$ **(3)**

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2. (a) $A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r$$ **(1)**

(b) given $\frac{dr}{dt} = \frac{1}{4}$.

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$r = 3$, $\quad \frac{dA}{dt} = (2\pi \times 3) \times \left(\frac{1}{4}\right) = \frac{3\pi}{2}$ cm^2 s^{-1} **(4)**

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3. (a) $1 + \frac{1}{2}(8x) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}(8x)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{3.2}$

$$= 1 + 4x - 8x^2 + 32x^3$$

(b) $-\frac{1}{8} < x < \frac{1}{8}$

(c) $(1+ax)(1+4x-8x^2+32x^3) = 1+4x-8x^2+3$

$4 + a = -8 + 4a$

$12 = 3a, \quad a = 4$

coef. of $x^3 = 32 - 8a = 32 - 32 = 0$

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4. (a) $t = 10, \quad m = 500\,e^{-0.2} = 409.4$ grams

(b) $m = 300, \quad 300 = 500\,e^{-0.02t}$

$$e^{-0.02t} = \frac{3}{5}$$

$$\ln e^{-0.02t} = \ln\left(\frac{3}{5}\right)$$

$$-0.02t = \ln\frac{3}{5}$$

$$t = 25.5 \text{ years}$$

(c) $\frac{dm}{dt} = 500 \times (-0.02)\,e^{-0.02t}$

when $t = 1$, $\quad \frac{dm}{dt} = 500 \times (-0.02) \times e^{-0.02} = -$

mass is decreasing at 9.8 g/year

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5. (a) (i) $\int (1 + e^x + e^{-x} + 1)\,dx = 2x + e^x - e^{-x} + c$ **(2)**

(ii) $\int (6x - 1)^{-\frac{1}{2}}\,dx = 2 \times \frac{1}{6}(6x-1)^{\frac{1}{2}} + c = \frac{1}{3}\sqrt{6x-1} + c$ **(2)**

(b) $\displaystyle\int_0^{\frac{\pi}{6}} x\cos x\ dx = \int_0^{\frac{\pi}{6}} x\frac{d}{dx}(\sin x)\ dx$

$$= \Big[x\sin x\Big]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \sin x\ dx$$

$$= \frac{\pi}{6}\cdot\frac{1}{2} - 0 + \Big[\cos x\Big]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$ **(5)**

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6. (a) $\dfrac{dy}{d\theta} = 2.2\cos\theta(-\sin\theta)$

$\dfrac{dx}{d\theta} = \cos\theta$

$\dfrac{dy}{dx} = \dfrac{-4\sin\theta\cos\theta}{\cos\theta} = -4\sin\theta$

at $\theta = \dfrac{\pi}{6}$, gradient of tangent $= -4\cdot\dfrac{1}{2} = -2$

at $\theta = \dfrac{\pi}{6}$, $x = \dfrac{1}{2}$, $y = 2\left(\dfrac{\sqrt{3}}{2}\right)^2 = \dfrac{3}{2}$

equation of tangent is $y - \dfrac{3}{2} = -2\left(x - \dfrac{1}{2}\right)$

$$2y + 4x = 5$$ **(5)**

(b) $y = 2\cos^2\theta = 2(1 - \sin^2\theta)$

$y = 2(1 - x^2)$

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7. (a) $\dfrac{1}{(y-1)y} = \dfrac{1}{y-1} - \dfrac{1}{y}$

(b) $\displaystyle\int\left(\dfrac{1}{y-1} - \dfrac{1}{y}\right)dy = \int \cos x\ dx$

$\ln(y-1) - \ln y = \sin x + c$

$y = 5,\quad x = 0,\qquad \ln 4 - \ln 5 = 0 + c$

$$\ln\frac{4}{5} = c$$

$\therefore\quad \ln\dfrac{y-1}{y} = \sin x + \ln\dfrac{4}{5}.$

$\ln\dfrac{y-1}{y} - \ln\dfrac{4}{5} = \sin x$

$\ln\dfrac{5(y-1)}{4y} = \sin x$

$\dfrac{5y-5}{4y} = e^{\sin x}$

$5y - 5 = 4y\,e^{\sin x}$

$y\left(5 - 4e^{\sin x}\right) = 5$

$$y = \dfrac{5}{5 - 4e^{\sin x}}$$

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8. (*a*) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

equation of AB is $r = \begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ **(3)**

(b) $\overrightarrow{AC} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ -7 \\ -3 \end{pmatrix} = 12 - 12 = 0$

so $12\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$ is perpendicular to AC.   **(2)**

(c) let $\angle BAC = \theta$

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = \left|\overrightarrow{AB}\right| \times \left|\overrightarrow{AC}\right| \cos\theta$$

$$\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} = \sqrt{2^2 + 3^2 + 1^2} \times \sqrt{1^2 + 4^2} \cos\theta$$

$$6 = \sqrt{14}\sqrt{17} \cos\theta$$

$$\cos\theta = \frac{6}{\sqrt{14}\sqrt{17}} \qquad \theta = 67°$$ **(4)**

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9. (*a*) At A $x \ln x = 0$

$\therefore$ $\ln x = 0$ $(x \neq 0)$ $x = 1$

(*b*) $\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x = 1 + \ln x$

at P $\frac{dy}{dx} = 0$, $\ln x = -1$

$x = e^{-1} = \frac{1}{e}$ and $y = \frac{1}{e} \ln e^{-1} = -\frac{1}{e}$

(c) area $= \displaystyle\int_{\frac{1}{e}}^{1} x \ln x \; dx = \int_{\frac{1}{e}}^{1} \ln x \frac{d}{dx}\left(\frac{x^2}{2}\right) dx$ (B

$= \left[\frac{x^2}{2} \ln x\right]_{\frac{1}{e}}^{1} - \int_{\frac{1}{e}}^{1} \frac{x^2}{2} \cdot \frac{1}{x} \; dx$

$= 0 - \frac{1}{2e^2} \ln e^{-1} - \left[\frac{x^2}{4}\right]_{\frac{1}{e}}^{1} = \frac{1}{2e^2} - \left[\frac{1}{4} - \frac{1}{4e}\right.$

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